West Virginia ACADEMY of SCIENCE Founded 1924

Original Research Paper

A Poynting Vector Analogue in Rotating Reference Frames

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Article history
Received: 23 February 2026
Revised: 18 April 2026
Accepted: 30 April 2026

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Abstract: The equation of motion of a particle of mass m at position $\boldsymbol{r}$ and moving with velocity $\dot{\boldsymbol{r}}$ in a non-inertial reference frame rotating with angular velocity $\boldsymbol{\omega}$ can be expressed in terms of mechanical electric $\boldsymbol{E}$ and magnetic $\boldsymbol{H}$ field analogues of those found in electromagnetic theory. In this paper these analogue fields are used to define a Poynting vector analogue field $\boldsymbol{S}$ associated with the motion of a particle in a rotating reference frame. $\boldsymbol{S}$ can be interpreted as a *force current vector field* and it is shown that the $\boldsymbol{S}$ vector field has a natural Helmholtz decomposition – the irrotational component being $\frac{1}{2}$ the previously identified centripetal force current vector field obtained from Ampere's law when applied to these analogue $\boldsymbol{E}$ and $\boldsymbol{H}$ fields. The Divergence Theorem and Stoke's Theorem are used to associate $\boldsymbol{S}$ with rotational power and rotational kinetic energy.

Keywords: force current; Poynting vector; analogues.

Introduction

Freeman Dyson reconstructed a proof (Dyson 1990) of a theorem shown to him by Richard Feynmann in 1948 that *if a non-relativistic quantum mechanical point particle of mass m satisfies canonical Cartesian position and momentum commutation relations and Newton's equation of motion, then there exist vector fields analogous to the electromagnetic electric and magnetic fields so that the associated force equation has the Lorentz form*. This result was extended by Richard Hughes (Hughes 1992) to non-relativistic classical particles. The motion of a particle in a non-inertial reference frame was used to illustrate this extension using associated mechanical $\boldsymbol{E}$ and $\boldsymbol{H}$ analogue vector fields.

In this paper Hughes' analogue $\boldsymbol{E}$ and $\boldsymbol{H}$ vector fields are used to define a *theoretical* analogue Poynting vector field $\boldsymbol{S}$ for a particle of mass m moving in a non-inertial reference frame. The properties of the $\boldsymbol{S}$ field are identified and discussed and the Divergence Theorem and Stoke's Theorem are used to relate $\boldsymbol{S}$ to rotational power and kinetic energy.

For the sake of clarity and precision, the vector calculus calculations in the following sections are performed in Cartesian coordinate presentations defined by the orthonormal vectors $\hat{\boldsymbol{\imath}}, \hat{\boldsymbol{\jmath}}$, and $\hat{\boldsymbol{k}}$. An Appendix containing definitions and Cartesian coordinate representations of vector quantities is provided for the reader's convenience.

Motion in a Non-inertial Reference Frame

Consider a reference frame whose origin R_0 executes a translational acceleration $\boldsymbol{a}$ and rotates with an angular velocity $\boldsymbol{\omega}$ relative to an inertial frame. If $\boldsymbol{r}$ is the position vector of a particle of mass m relative to R_0 and the particle is moving with velocity $\dot{\boldsymbol{r}}$, then its equation of motion is (Landau and Lifshitz 1976)

$$m\ddot{\boldsymbol{r}} = -m\boldsymbol{a} + m\boldsymbol{r} \times \dot{\boldsymbol{\omega}} + 2m\dot{\boldsymbol{r}} \times \boldsymbol{\omega} + m\boldsymbol{\omega} \times (\boldsymbol{r} \times \boldsymbol{\omega}). \quad (1)$$

Here, the second, third, and fourth terms on the right-hand side of the last equation are the Euler, Coriolis, and centrifugal inertial forces, respectively. The physical force (that force acting on m in the inertial frame) is set to zero here since the interest is strictly in the electromagnetic analogue.

It is found that (1) may be written in Lorentz form as (Hughes 1992)

$$m\ddot{\boldsymbol{r}} = \boldsymbol{E}(\boldsymbol{r},t) + \dot{\boldsymbol{r}} \times \boldsymbol{H}(\boldsymbol{r},t),$$

where (refer to Appendix)

$$\boldsymbol{E}(\boldsymbol{r},t) = m\boldsymbol{\omega} \times (\boldsymbol{r} \times \boldsymbol{\omega}) - m\dot{\boldsymbol{\omega}} \times \boldsymbol{r}$$
$$= m\{(\omega^2 x + \dot{\omega} y)\hat{\boldsymbol{\imath}} + (\omega^2 y - \dot{\omega} x)\hat{\boldsymbol{\jmath}}\} \quad (2)$$

and

$$\boldsymbol{H}(\boldsymbol{r},t) = 2m\boldsymbol{\omega}$$
$$= 2m\omega\hat{\boldsymbol{k}} \quad (3)$$

are the mechanical analogues of the electromagnetic field vectors (here, $\boldsymbol{a} = \boldsymbol{0}$ is used for simplification and does not change the main results).

The Analogue Poynting Vector

Using the form of the electromagnetic Poynting vector as a model, define the analogue mechanical Poynting vector field as the vector product

$$\boldsymbol{S} \equiv \frac{1}{2m}(\boldsymbol{E} \times \boldsymbol{H}). \quad (4)$$

Substituting (2) and (3) into this and rearranging yields

$$\boldsymbol{S} = \boldsymbol{V} + \left(\tfrac{1}{2}\boldsymbol{\mathcal{J}}\right), \quad (5)$$

where

$$\boldsymbol{V} = -m\omega^2 \boldsymbol{v} \quad (6)$$

and

$$\boldsymbol{\mathcal{J}} = -2m\omega\dot{\omega}\boldsymbol{r}. \quad (7)$$

Clearly, $\boldsymbol{S}$ is the sum of two force current vector fields: $\boldsymbol{V}$ is a new linear momentum $m\boldsymbol{v}$ current field induced by $\boldsymbol{S}$ and $\boldsymbol{\mathcal{J}}$ is the previously identified centripetal force $\boldsymbol{C}$ current vector field (Parks 2025). Here, use has been made of the fact that $\boldsymbol{v} = \boldsymbol{\omega} \times \boldsymbol{r}$, and, since motion occurs in the $\hat{\boldsymbol{\imath}}\,\hat{\boldsymbol{\jmath}}$ plane, $\boldsymbol{r} = x\hat{\boldsymbol{\imath}} + y\hat{\boldsymbol{\jmath}}$. Since $\boldsymbol{\mathcal{J}} \propto -\boldsymbol{r}$ and $\boldsymbol{V} \propto -\boldsymbol{v}$, then $\boldsymbol{\mathcal{J}}$ is directed along $\boldsymbol{r}$ towards the axis of rotation and $\boldsymbol{V}$ is in the $\hat{\boldsymbol{\imath}}\,\hat{\boldsymbol{\jmath}}$ plane opposite the direction of m's velocity.

The physical unit associated with the $\boldsymbol{V}$ and $\boldsymbol{\mathcal{J}}$ fields is *mass* × *length / time* ^{3}. Because this unit is the same as *force / time*, $\boldsymbol{S}$, $\boldsymbol{V}$, and $\boldsymbol{\mathcal{J}}$ are called *force current vector fields* (although this unit is the same as mechanical jerk, it is not jerk because it is not the time derivative of force).

Properties of the *S* Vector Field

Recall that the Helmholtz decomposition theorem states that certain differentiable vector fields can be decomposed into a divergence free (solenoidal) vector field and a curl free (irrotational) vector field. The following theorem identifies such a natural decomposition of the $\boldsymbol{S}$ field that follows in a straightforward manner from substitution of (2) and (3) into (4).

Theorem 1. *Equation (5) is a Helmholtz decomposition of the $\boldsymbol{S}$ vector field.*

Proof.
(i) $\boldsymbol{V}$ and $\boldsymbol{\mathcal{J}}$ are orthogonal force current vector fields because $\boldsymbol{V} \cdot \boldsymbol{\mathcal{J}} = 2m^2\omega^3\dot{\omega}\boldsymbol{v} \cdot \boldsymbol{r} = 2m^2\omega^3\dot{\omega}[-\omega(y\hat{\boldsymbol{\imath}} - x\hat{\boldsymbol{\jmath}}) \cdot (x\hat{\boldsymbol{\imath}} + y\hat{\boldsymbol{\jmath}}) = -2m^2\omega^4\dot{\omega}(yx - xy) = -2m^2\omega^4\dot{\omega}(0) = 0.$

(ii) $\boldsymbol{V}$ is a solenoidal field because $\boldsymbol{\nabla} \cdot \boldsymbol{V} = \left(\hat{\boldsymbol{\imath}}\,\partial_x + \hat{\boldsymbol{\jmath}}\,\partial_y + \hat{\boldsymbol{k}}\,\partial_z\right) \cdot [-m\omega^2\boldsymbol{v}] = -m\omega^2\left(\hat{\boldsymbol{\imath}}\,\partial_x + \hat{\boldsymbol{\jmath}}\,\partial_y + \hat{\boldsymbol{k}}\,\partial_z\right) \cdot \boldsymbol{v} = -m\omega^2\left(\hat{\boldsymbol{\imath}}\,\partial_x + \hat{\boldsymbol{\jmath}}\,\partial_y + \hat{\boldsymbol{k}}\,\partial_z\right) \cdot (\boldsymbol{\omega} \times \boldsymbol{r}) = m\omega^3\left(\hat{\boldsymbol{\imath}}\,\partial_x + \hat{\boldsymbol{\jmath}}\,\partial_y + \hat{\boldsymbol{k}}\,\partial_z\right) \cdot (y\hat{\boldsymbol{\imath}} - x\hat{\boldsymbol{\jmath}}) = m\omega^3\left(\partial_x y - \partial_y x\right) = m\omega^3(0 - 0) = 0.$

(iii) $\frac{1}{2}\boldsymbol{\mathcal{J}}$ is an irrotational field because $\boldsymbol{\nabla} \times \frac{1}{2}\boldsymbol{\mathcal{J}} = -m\omega\dot{\omega}\boldsymbol{\nabla} \times \boldsymbol{r} = -m\omega\dot{\omega}\left[\hat{\boldsymbol{\imath}}\left(\partial_y 0 - \partial_z y\right) + \hat{\boldsymbol{\jmath}}(\partial_z x - \partial_x 0) + \hat{\boldsymbol{k}}\left(\partial_x y - \partial_y x\right)\right] = -m\omega\dot{\omega}\left[\hat{\boldsymbol{\imath}}(0 - 0) + \hat{\boldsymbol{\jmath}}(0 - 0) + \hat{\boldsymbol{k}}(0 - 0)\right] = \boldsymbol{0}$. ■

Since the divergence and curl of $\boldsymbol{S}$ are required in the following section, it is necessary to determine - in the next two theorems - the divergence of the irrotational field and the curl of the solenoidal field.

Theorem 2. *The divergence of the irrotational field is* $\frac{1}{2}\boldsymbol{\nabla} \cdot \boldsymbol{\mathcal{J}} = -2m\omega\dot{\omega}$.

Proof.

$$\tfrac{1}{2}\boldsymbol{\nabla} \cdot \boldsymbol{\mathcal{J}} = -m\omega\dot{\omega}\left(\hat{\boldsymbol{\imath}}\,\partial_x + \hat{\boldsymbol{\jmath}}\,\partial_y + \hat{\boldsymbol{k}}\,\partial_z\right) \cdot (x\hat{\boldsymbol{\imath}} + y\hat{\boldsymbol{\jmath}})$$
$$= -m\omega\dot{\omega}\left(\partial_x x + \partial_y y\right)$$
$$= -m\omega\dot{\omega}(1 + 1)$$
$$= -2m\omega\dot{\omega}. \quad ■$$

Theorem 3. *The curl of the solenoidal field is* $\boldsymbol{\nabla} \times \boldsymbol{V} = -2m\omega^3\hat{\boldsymbol{k}}$.

Proof.

$$\begin{aligned}\nabla\times\boldsymbol{V} &= -m\omega^2\nabla\times\boldsymbol{v}\\ &= m\omega^3\left[\hat{\boldsymbol{\imath}}\left(\partial_y 0-\partial_z(-x)\right)+\hat{\boldsymbol{\jmath}}(\partial_z y-\partial_x 0)\right.\\ &\qquad \left.+\hat{\boldsymbol{k}}\left(\partial_x(-x)-\partial_y y\right)\right]\\ &= m\omega^3[\hat{\boldsymbol{\imath}}0+\hat{\boldsymbol{\jmath}}0-\hat{\boldsymbol{k}}2]\\ &= -2m\omega^3\hat{\boldsymbol{k}}. \quad \blacksquare\end{aligned}$$

It follows from the last three theorems that

$$\nabla\cdot\boldsymbol{S}=\tfrac{1}{2}\nabla\cdot\boldsymbol{\mathcal{J}}=-2m\omega\dot{\omega} \tag{8}$$

and

$$\nabla\times\boldsymbol{S}=\nabla\times\boldsymbol{V}=-2m\omega^3\hat{\boldsymbol{k}}. \tag{9}$$

Thus, the divergence and curl of the $\boldsymbol{S}$ field result entirely from the divergence of $\boldsymbol{\mathcal{J}}$ and the curl of $\boldsymbol{V}$, repectively.

The Power Theorems

Let $\boldsymbol{r}$ be the position of *m* in the $\hat{\boldsymbol{\imath}}\,\hat{\boldsymbol{\jmath}}$ plane and let *A* be a circle of radius *r* in the $\hat{\boldsymbol{\imath}}\,\hat{\boldsymbol{\jmath}}$ plane centered at its origin.

Theorem 4. *There is an influx across the circle's boundary ∂A into A proportional to the rotational power $\wp$.*

Proof.
The two dimensional divergence theorem (Fulks 1964) relates the integral of $\nabla\cdot\boldsymbol{S}$ extended over A to a contour integral extended along the boundary ∂A of A according to

$$\oiint_A \nabla\cdot\boldsymbol{S}\,dxdy=\oint_{\partial A}\boldsymbol{S}\cdot\hat{\boldsymbol{n}}\,d\ell,$$

where $\hat{\boldsymbol{n}}$ is the outward unit normal vector for ∂A and $d\ell$ is the differential element of arc length along ∂A. Substituting (8) into the left hand side of the last equation yields

$$\begin{aligned}\oiint_A \nabla\cdot\boldsymbol{S}\,dxdy &= \oiint_A \nabla\cdot\tfrac{1}{2}\boldsymbol{\mathcal{J}}\,dxdy\\ &= -2m\omega\dot{\omega}\oiint_A dxdy\\ &= -2m\omega\dot{\omega}(\pi r^2)\\ &= -2\pi I\omega\dot{\omega}\\ &= -2\pi\tau\omega\\ &= -2\pi\wp.\end{aligned}$$

Here, $I=mr^2$ is the moment of inertia, $\tau=I\dot{\omega}$ is the torque, and $\wp$ is the associated rotational power. Transitivity of equality yields

$$\oint_{\partial A}\boldsymbol{S}\cdot\hat{\boldsymbol{n}}\,d\ell=-2\pi\wp. \tag{10}$$

This integral corresponds to a flux of rotational power across ∂A and the negative sign indicates the flux is into A. ■

Theorem 5. *The vorticity of $\boldsymbol{S}$ is clockwise around $\hat{\boldsymbol{k}}$ and proportional to ωK, where K is the rotational kinetic energy.*

Proof.
The two dimensional Stoke's theorem (Fulks 1964) relates the integral of $\nabla\times\boldsymbol{S}$ extended over A to a contour integral extended along the boundary ∂A of A according to

$$\oiint_A (\nabla\times\boldsymbol{S})\cdot\hat{\boldsymbol{k}}dA=\oint_{\partial A}\boldsymbol{S}\cdot\hat{\boldsymbol{t}}\,d\ell,$$

where $\hat{\boldsymbol{t}}$ is the tangent vector along ∂A. The right hand side of this equation is called the circulation of $\boldsymbol{S}$ around ∂A and measures the vorticity (rotation) associated with $\boldsymbol{S}$. Substituting (9) into the left hand side of the last equation yields

$$\begin{aligned}\oiint_A (\nabla\times\boldsymbol{S})\cdot\hat{\boldsymbol{k}}\,dxdy &= \oiint_A (\nabla\times\boldsymbol{V})\cdot\hat{\boldsymbol{k}}\,dxdy\\ &= -2m\omega^3\hat{\boldsymbol{k}}\cdot\hat{\boldsymbol{k}}\oiint_A dxdy\\ &= -2m\omega^3(\pi r^2)\\ &= -2\pi I\omega^3\\ &= -4\pi\omega(\tfrac{1}{2}I\omega^2)\\ &= -4\pi\omega K.\end{aligned}$$

Transitivity of equality yields the circulation

$$\oint_{\partial A}\boldsymbol{S}\cdot\hat{\boldsymbol{t}}\,d\ell=-4\pi\omega K. \tag{11}$$

Since this result is negative, the rotation is clockwise around $\hat{\boldsymbol{k}}$. ■

(This theorem is referred to as a power theorem because the physical unit of ωK is *energy / time = power*.)

Discussion

While the electromagnetic Poynting vector gives the direction and rate of electromagnetic energy flow per unit area at a point in space, the *theoretical* analogue mechanical Poynting vector introduced here gives the direction and magnitude of force current at a point in the $\hat{\boldsymbol{\imath}}\,\hat{\boldsymbol{\jmath}}$ plane of a rotating reference frame. The word "theoretical" is emphasized here because force currents have not been observed in rotating reference frames.

Using (5) and the fact that $\boldsymbol{V}$ and $\boldsymbol{\mathcal{J}}$ are orthogonal, the magnitude S of a Poynting vector $\boldsymbol{S}$ is given by

$$S=(\boldsymbol{S}\cdot\boldsymbol{S})^{\frac{1}{2}}=\left(\boldsymbol{V}\cdot\boldsymbol{V}+\frac{1}{4}\boldsymbol{\mathcal{J}}\cdot\boldsymbol{\mathcal{J}}\right)^{\frac{1}{2}}.$$

After substituting (6), (7), $v=\omega r$, and simplifying yields

$$S=m\omega r(\omega^4+\dot{\omega}^2)^{\frac{1}{2}}. \tag{12}$$

The angle θ from m relative to $\boldsymbol{r}$ (obtained from application of the "tip-to-tail" method to the orthogonal vectors (6) and (7)**)** defines the direction of the associated force current as

$$\theta = \tan^{-1}\left(\frac{\omega^2}{2\dot{\omega}}\right). \quad (13)$$

Observe from (7), (12), and (13) that when $\dot{\omega} = 0$, – as required - $\boldsymbol{\mathcal{J}} = \mathbf{0}$, $S = m\omega^3 r = m\omega^2 v = (\boldsymbol{V}\cdot\boldsymbol{V})^{\frac{1}{2}} = V$, and $\theta = \frac{\pi}{2}$.

It should be noted that because (5) is a Helmholtz decomposition, $\boldsymbol{S}$ can be expressed in terms of a scalar potential φ and a vector potential $\boldsymbol{A}$ as

$$\boldsymbol{S} = -\boldsymbol{\nabla}\varphi + \boldsymbol{\nabla}\times\boldsymbol{A},$$

where

$$-\boldsymbol{\nabla}\varphi = \frac{1}{2}\boldsymbol{\mathcal{J}} = -m\omega\dot{\omega}\boldsymbol{r} \quad (14)$$

and

$$\boldsymbol{\nabla}\times\boldsymbol{A} = -m\omega^2\boldsymbol{v}. \quad (15)$$

The following two theorems identify two such potentials.

Theorem 6. *The scalar potential* $\varphi = \frac{1}{2}m\frac{\dot{\omega}}{\omega}(\boldsymbol{\omega}\times\boldsymbol{r})^2$ *satisfies equation (14).*

Proof.

$$\begin{aligned}\boldsymbol{\nabla}\varphi &= \frac{1}{2}m\frac{\dot{\omega}}{\omega}\boldsymbol{\nabla}(\boldsymbol{\omega}\times\boldsymbol{r})^2\\ &= \frac{1}{2}m\frac{\dot{\omega}}{\omega}\omega^2(\hat{\boldsymbol{\imath}}\partial_x + \hat{\boldsymbol{\jmath}}\partial_y + \hat{\boldsymbol{k}}\partial_z)(\hat{\boldsymbol{\imath}}y - \hat{\boldsymbol{\jmath}}x)\cdot(\hat{\boldsymbol{\imath}}y - \hat{\boldsymbol{\jmath}}x)\\ &= \frac{1}{2}m\frac{\dot{\omega}}{\omega}\omega^2(\hat{\boldsymbol{\imath}}\partial_x + \hat{\boldsymbol{\jmath}}\partial_y + \hat{\boldsymbol{k}}\partial_z)(y^2 + x^2)\\ &= \frac{1}{2}m\frac{\dot{\omega}}{\omega}\omega^2 2(\hat{\boldsymbol{\imath}}x + \hat{\boldsymbol{\jmath}}y)\\ &= m\omega\dot{\omega}\boldsymbol{r}.\end{aligned}$$

Thus,

$$-\boldsymbol{\nabla}\varphi = -m\omega\dot{\omega}\boldsymbol{r}$$

and (14) is satisfied. ■

Theorem 7. *The vector potential* $\boldsymbol{A} = \frac{1}{2}m\omega^3 r^2\hat{\boldsymbol{k}}$ *satisfies equation (15).*

Proof.
From (15) and the Appendix,

$$\boldsymbol{\nabla}\times\boldsymbol{A} = -m\omega^2\boldsymbol{v} = -m\omega^2(\boldsymbol{\omega}\times\boldsymbol{r})$$

$$\hat{\boldsymbol{\imath}}(\partial_y A_z - \partial_z A_y) + \hat{\boldsymbol{\jmath}}(\partial_z A_x - \partial_x A_z) + \hat{\boldsymbol{k}}(\partial_x A_y - \partial_y A_x) = m\omega^3(\hat{\boldsymbol{\imath}}y - \hat{\boldsymbol{\jmath}}x).$$

Equating the coefficients of the unit vectors yields

$$\partial_y A_z - \partial_z A_y = m\omega^3 y$$
$$\partial_z A_x - \partial_x A_z = -m\omega^3 x$$

and

$$\partial_x A_y - \partial_y A_x = 0.$$

Now choose

$$A_x = 0 = A_y \quad (16)$$

in which case the last equation is satisfied so that

$$\partial_y A_z = m\omega^3 y$$

and

$$\partial_x A_z = m\omega^3 x.$$

By inspection, it can be seen that

$$A_z = \frac{1}{2}m\omega^3(x^2 + y^2) = \frac{1}{2}m\omega^3 r^2$$

and the theorem is proved. ■

These potentials are not unique (for example, replace (16) with $A_x = zx$ and $A_y = zy$ in which case $A_z = \frac{1}{2}(1 + m\omega^3)(x^2 + y^2)$. Although these potentials appear to provide no useful computational simplification or any additional insight into the properties of the $\boldsymbol{S}$ field beyond what is provided by $\boldsymbol{\mathcal{J}}$ and $\boldsymbol{V}$, they are introduced here for the sake of completeness.

Closing Remarks

This paper has introduced a theoretical mechanical Poynting vector field $\boldsymbol{S}$ associated with rotating reference frames. This force current field will theoretically always be present as long as $\omega \neq 0$. $\boldsymbol{S}$ has a natural Helmholtz decomposition. The solenoid field is opposite the direction of motion of the mass m and is reminiscent of thrust. The irrotational field is directed radially from m towards the center of rotation and was previously discovered independent of $\boldsymbol{S}$ via application of Ampere's law to the analogue mechanical electric and magnetic fields given by (2) and (3) above (Parks 2025). Associated scalar and vector potentials were also identified.

Perhaps the most unexpected result presented in this paper is the fact that application of the Divergence and Stoke's theorems to $\boldsymbol{S}$ relate rotational power and rotational kinetic energy to the contour integrals (10) and (11).

The question remains "how can the centripetal force vector $\boldsymbol{C}$ and the linear momentum $-m\boldsymbol{v}$ vector be obtained at some time $t^{'}$ from the current vectors $\frac{1}{2}\boldsymbol{\mathcal{J}}$ and $\boldsymbol{V}$ at time t, respectively"? This could be accomplished formally via transfer functions $\phi(t,t^{'})$ and $\psi(t,t^{'})$ according to

$$\boldsymbol{C}(t') = \int_{-\infty}^{+\infty} \phi(t,t')\tfrac{1}{2}\boldsymbol{\mathcal{J}}(t)\,dt$$

and

$$-m\boldsymbol{v}(t') = \int_{-\infty}^{+\infty} \psi(t,t')\,\boldsymbol{V}(t)\,dt$$

(clearly, when $\boldsymbol{r}$ is time dependent, then so are the vectors $\frac{1}{2}\boldsymbol{\mathcal{J}}$, $\boldsymbol{V}$, $\boldsymbol{C}$, and $-m\boldsymbol{v}$). Obvious example candidates for these transfer functions are

$$\phi(t,t') = \frac{\omega}{\dot{\omega}}\,\delta(t-t')$$

and

$$\psi(t,t') = \frac{1}{\omega^2}\,\delta(t-t'),$$

where $\delta(t-t')$ is the Dirac delta function which has the property

$$\int_{-\infty}^{+\infty} f(t)\,\delta(t-t')dt = f(t')\,.$$

Appendix

$$\hat{\boldsymbol{\imath}}\cdot\hat{\boldsymbol{\imath}} = \hat{\boldsymbol{\jmath}}\cdot\hat{\boldsymbol{\jmath}} = \hat{\boldsymbol{k}}\cdot\hat{\boldsymbol{k}} = 1$$

$$\hat{\boldsymbol{\imath}}\cdot\hat{\boldsymbol{\jmath}} = \hat{\boldsymbol{\imath}}\cdot\hat{\boldsymbol{k}} = \hat{\boldsymbol{\jmath}}\cdot\hat{\boldsymbol{k}} = 0$$

$$\partial_\ell \equiv \frac{\partial}{\partial \ell}\,, \ell = x,y,z$$

$$\boldsymbol{\nabla} = \hat{\boldsymbol{\imath}}\partial_x + \hat{\boldsymbol{\jmath}}\partial_y + \hat{\boldsymbol{k}}\partial_z$$

$$\boldsymbol{a} = \hat{\boldsymbol{\imath}}a_x + \hat{\boldsymbol{\jmath}}a_y + \hat{\boldsymbol{k}}a_z;\ a_\ell\ constant, \ell = x,y,z$$

$$\boldsymbol{r} = \hat{\boldsymbol{\imath}}x + \hat{\boldsymbol{\jmath}}y + \hat{\boldsymbol{k}}z$$

$$\boldsymbol{\omega} = \hat{\boldsymbol{k}}\omega, \omega\ constant$$

$$\dot{\boldsymbol{\omega}} = \hat{\boldsymbol{k}}\dot{\omega}, \dot{\omega}\ constant$$

$$\dot{\boldsymbol{r}} \equiv \frac{d\boldsymbol{r}}{dt} = \hat{\boldsymbol{\imath}}\dot{x} + \hat{\boldsymbol{\jmath}}\dot{y} + \hat{\boldsymbol{k}}\dot{z}$$

$$\ddot{\boldsymbol{r}} \equiv \frac{d^2\boldsymbol{r}}{dt^2}$$

$$\boldsymbol{\omega} = \hat{\boldsymbol{k}}\omega, \omega\ constant$$

$$\dot{\boldsymbol{\omega}} \equiv \frac{d\boldsymbol{\omega}}{dt} = \hat{\boldsymbol{k}}\dot{\omega}, \dot{\omega}\ constant$$

$$\boldsymbol{C}\times\boldsymbol{D} = \hat{\boldsymbol{\imath}}(c_y d_z - c_z d_y) + \hat{\boldsymbol{\jmath}}(c_z d_x - c_x d_z) + \hat{\boldsymbol{k}}(c_x d_y - c_y d_x)$$

$$\begin{aligned}\boldsymbol{r}\times\boldsymbol{\omega} &= \hat{\boldsymbol{\imath}}(y\omega - z\cdot 0) + \hat{\boldsymbol{\jmath}}(z\cdot 0 - x\cdot\omega) + \hat{\boldsymbol{k}}(x\cdot 0 - y\cdot 0)\\ &= \hat{\boldsymbol{\imath}}y\omega - \hat{\boldsymbol{\jmath}}x\omega + \hat{\boldsymbol{k}}(0)\\ &= \omega(\hat{\boldsymbol{\imath}}y - \hat{\boldsymbol{\jmath}}x)\end{aligned}$$

$$\begin{aligned}\boldsymbol{\omega}\times\boldsymbol{r} &= -\boldsymbol{r}\times\boldsymbol{\omega}\\ &= \omega(\hat{\boldsymbol{\jmath}}x - \hat{\boldsymbol{\imath}}y)\end{aligned}$$

$$\begin{aligned}\boldsymbol{\omega}\times(\boldsymbol{r}\times\boldsymbol{\omega}) &= \hat{\boldsymbol{\imath}}[0\cdot 0 - \omega\cdot(-\omega x)] + \hat{\boldsymbol{\jmath}}[\omega\cdot(\omega y) - 0\cdot 0] + \hat{\boldsymbol{k}}[0\cdot(-\omega x) - 0\cdot\omega y]\\ &= \hat{\boldsymbol{\imath}}\omega^2 x + \hat{\boldsymbol{\jmath}}\omega^2 y + \hat{\boldsymbol{k}}(0)\\ &= \omega^2(\hat{\boldsymbol{\imath}}x + \hat{\boldsymbol{\jmath}}y)\end{aligned}$$

$$\begin{aligned}\dot{\boldsymbol{\omega}}\times\boldsymbol{r} &= \hat{\boldsymbol{\imath}}(0\cdot z - \dot{\omega}\cdot y) + \hat{\boldsymbol{\jmath}}(\dot{\omega}\cdot x - 0\cdot z) + \hat{\boldsymbol{k}}(0\cdot y - 0\cdot x)\\ &= \hat{\boldsymbol{\imath}}(-\dot{\omega}\cdot y) + \hat{\boldsymbol{\jmath}}(\dot{\omega}\cdot x) + \hat{\boldsymbol{k}}(0)\\ &= -\dot{\omega}(\hat{\boldsymbol{\imath}}y - \hat{\boldsymbol{\jmath}}x)\end{aligned}$$

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Parks, A. 2025. Electromagnetic Analogues in Rotating Reference Frames. *Proceedings of the West Virginia Academy of Science,* 97(1): 79-84.